# Student's Note - Week 2 - Formulas for Sequences

This lesson has two parts - concept and problem.

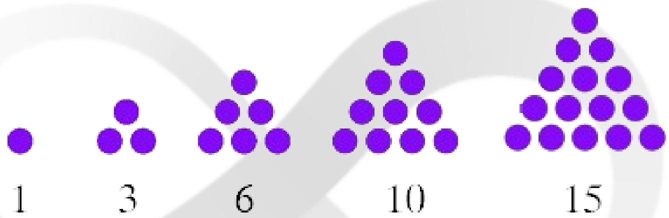
- Concept session - 45 minutes
- Activity I - 10 minutes
- Activity II - 10 minutes
- End of class quiz - 10 minutes

## Concept - Formulas for Sequences

**Deduce the formula for $n^{th}$ term**

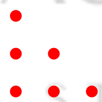
**Deduce the formula for nth term - *Triangular sequence***

We learnt the ***triangular sequence*** last week. The sequence is $1, 3, 6, 10, 15, 21, \ldots$. This can also be demonstrated in the following way

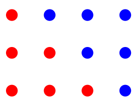



Clearly, the serial number of the term in the ***triangular sequence*** denotes the number of layers in the triangle. That implies that the $10^{th}$ term will have ***ten*** layers. So, the number of dots present there in the triangle is $1 + 2 + 3 + 4 + ldots + 9 + 10 = 55$. It is quiet easy to understand, but the problem will appear if we need to find some larger term like $100^{th}$ term. Let's try to get the general term of the ***triangular sequence*** as well.

Lets try to get a pattern :



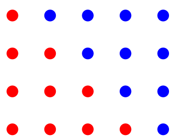
This is the third term of the ***triangular sequence***. If we add another congruent triangle here it should look like this :



This is forming a `rectangle`.So, the total number of dots is actually the half of the area of the rectangle.

$$\frac{4 \times 3}{2}$$

Similarly, if we try for the next order then,



Total number of dots here is

$$\frac{5 \times 4}{2}$$

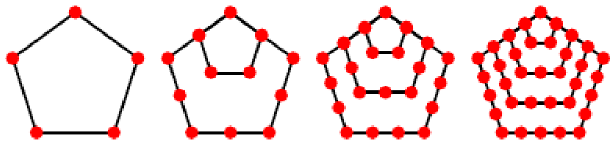
That means ***the number of dots in a triangle is the half of the product of the serial number of the term and the next number .*** That leads us to conclude that the $n^{th}$ term of the `triangular sequence`, which is

$$\boxed{\frac{n \times (n+1)}{2}}$$

So, the $100^{th}$ term of the sequence is

$$\frac{100 \times (100+1)}{2} = \frac{100 \times 101}{2} = 50 \times 101 = \boxed{5050}$$

**Deduce the formula for nth term - Pentagonal Numbers**



Pentagonal numbers numbers from $P_2$ to $P_5$.

Here is the figure for the Pentagonal number. Can you find out the next term from this figure? Can we try to find a general formula for nth term for any such values of n?

**Concept - Recurrence Relation**

What is recurrence relation in sequence? Explain with the following example.

Suppose the first term of a sequence is 2 and the following rule is satisfied by every other term of the sequence: every term is double the previous term. Can you write down a symbolic expression for this sentence?

$$Term_n = 2 \times Term_{n-1}$$

Can you guess a direct formula for this sequence $T_n = 2^n$

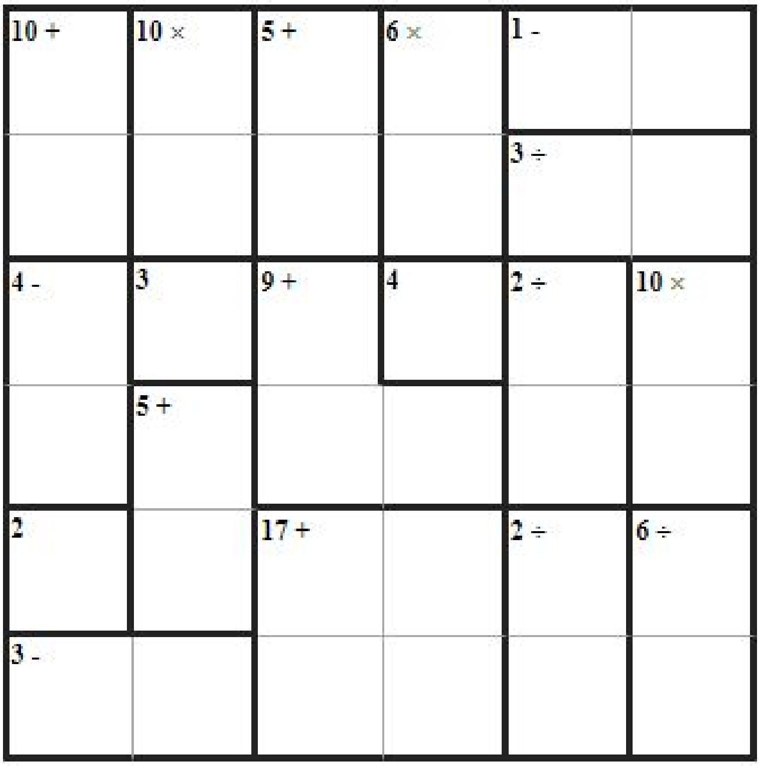
**Concept - Fibonacci Sequence**

Find the $n^{th}$ term for the Fibonacci sequence.

**Concept - Finding the sequence**

1) Find the $n^{th}$ term for the even number sequence.
2) Find the $n^{th}$ term for the odd number sequence.

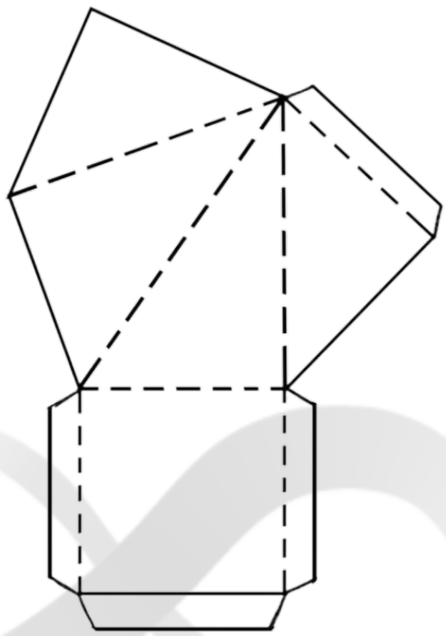
## Activity I - 10 minutes



## Activity II - 10 minutes

**Paper model of a stable pyramid(not with regular polygons)**

- To construct the pyramid we will use the following shape.



- Through the dark line we will cut the page and will fold it through the dotted lines.
- After the folding we will `glue` the paper where ever you need.
- Following this method we can have a nice stable ***pyramid***.

**Mixed Problems**

**Problem 1**

Three water pipes can be used to fill a water tank. The first pipe by itself takes 8 hours to fill the tank, the second pipe by itself takes 12 hours to fill the tank and the third pipe by itself takes 24 hours to fill the tank. How long would it take to fill the tank from empty if all pipes were used at the same time?

(A) 2hrs (B) 3hrs (C) 6hrs (D) 5hrs (E) 4hrs

**Problem 2**

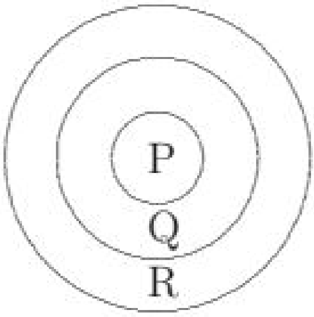
The average age of the 11 players in the Australian soccer team is 22.

One player got a red card and had to leave the field. Then, the average age of the remaining 10 players on the field was 21 . How old was the player with the red card?

(A) 21 (B) 31 (C) 22 (D) 32 (E) 24

**Problem 3**

Anne designs the dart board shown, where she scores $P$ points in the centre circle, $Q$ points in the next ring and $R$ points in the outer ring. She throws three darts in each turn. In her first turn, she gets two darts in ring $Q$ and one in ring $R$ and scores 10 points. In her second turn, she gets two in circle $P$ and one in ring $R$ and scores 22 points. In her next turn, she gets one dart in each of the regions. How many points does she score?






(A) 12 (B) 13 (C) 15 (D) 16 (E) 18

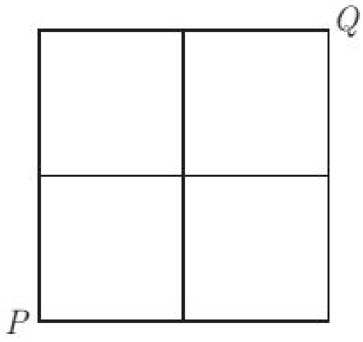
**Problem 4**

Paradise is a circular island. There are 4 villages on the island at the N, E, S and SE points. Each village has fishing rights for a single continuous strip of coastline, which must include the village plus 3 km either side. If the fishing rights are distributed as evenly as possible between the four villages under the above rules, and the difference between the length of coastline fished by the village at N and the village at SE is as small as possible, then this difference, in kilometres, is

(A) 6 (B) 9 (C) 12 (D) 15 (E) 36

**Problem 5**

A town centre has a series of roads which form a two by two square as shown. On any particular journey from $P$ to $Q$, I may not drive down the same section of road twice, though I may cross any intersection more than once. How many different journeys are there from $P$ to $Q$ ?




(A) 6 (B) 10 (C) 12 (D) 14 (E) 16