Student's Note - Week 1 - Cryptarithmetic - Vigenère cipher

This lesson has two parts - concept and problem.

- Concept session - 40 minutes
- Activity I - 10 minutes
- Activity II - 10 minutes
- Problem Solving - 15 minutes

Concept - Cryptarithmetic - Vigenère cipher - 40 minutes

Cryptarithmetic

Cryptarithmetic is a type of mathematical puzzle where letters or symbols are substituted for digits in an arithmetic equation. The objective of the puzzle is to find the numerical values that correspond to each letter so that when the equation is correctly solved, it satisfies the mathematical rules.

Cryptarithmetic puzzles are significant in the field of number theory as they help develop problem-solving skills and logical reasoning and also provide an interesting way to explore the properties of numbers and equations.

Example: Let's understand it with a simple example

$$A + T = TO$$

Every different letter represents a different number. So, A, T, and O represent a different number. To solve this puzzle we will use a few interesting observations and results which we will discuss in detail while we will learn ***How to solve Cryptarithmetic puzzles***.

If we add any two numbers from 0 to 9, the maximum sum we can get is 18, that too when we add 9 to another 9. Other than that the maximum we can get is $8+9 = \boxed{17}$

That makes us to have a conclusion that the maximum ***carry*** would be 1. That means the T of TO is 1.

$\therefore$ Sum of A and 1 is a two-digit number. Only possible case is $\boxed{A = 9}$.

That completes the puzzle : $\boxed{A = 1, T = 1, O = 0}$

How to solve Cryptarithmetic puzzles

The most interesting part of the Cryptarithmetic puzzles is to solve them. There are several ways to solve it, you can use any method that is convenient to you. We will understand the method of solving it with an example.

Problem 1 (NMTC '19)

If A, B, C, D, and E are distinct digits then find E.

$$\begin{array}{cccc} & A & B & C \\ + & C & B & A \\ \hline D & E & D & D \end{array}$$

Solution : The first thing we should keep in mind is that we should always consider the ***carry*** over.

From that carry over it's very to conclude that $\boxed{D = 1}$.

From the one's place, we get $A + C = D = 1$ but from the hundred's we get $A + C = E$. That is only possible if there is some carry from $B + B = D$. That means $B + B = 11$

$\therefore C + A + 1 = E \Rightarrow D + 1 = E \Rightarrow E = 1 + 1 \Rightarrow \boxed{E = 2}$ Although it is not required to find still let's try to find out about the others.

From ten's place, we can find that $B + B = 1$ but we have already figured out that there is a carry of 1 in the hundred's. That means $B + B = 11$. But it isn't possible that two same numbers are added to 11, which implies there must be a carry from the ones.

$\therefore B + B = 10 \Rightarrow \boxed{B = 5}$.

Again, $C + A = 11$ there would be multiple possible pairings exist for that: (2,9), (3,8), (4,7), (5,6). As 2 is already used that number cannot happen again.

Problem 2

Find the number represented by each number.

$$\begin{array}{cccccc} & S & E & N & D \\ + & M & O & R & E \\ \hline M & O & N & E & Y \end{array}$$

Solution : This is a very interesting problem. Try to maintain two things while solving such a puzzle: First, remember which numbers are already decoded and which are yet to. Second, make the same structure with the blank places like the puzzle.

$$\begin{array}{cccccc} & \square & \square & \square & \square \\ + & \square & \square & \square & \square \\ \hline \square & \square & \square & \square & \square \end{array}$$

As we discussed the only number that can be in the carry is 1 which means $\boxed{M = 1}$.

$$\begin{array}{cccccc} & \square & \square & \square & \square \\ + & \boxed{1} & \square & \square & \square \\ \hline \boxed{1} & \square & \square & \square & \square \end{array}$$

Now let's try to decode the others. Let's try to decode S.

$S + 1 = 10 + \text{O}$ as they are making 1 carry. As of now let's assume $S = 9$ if we get any contradiction we will change it to 8, in case there is a carry from $E + O$.

So, $S + 1 = 9 + 1 = 10 \Rightarrow \boxed{\text{O} = 0}$

$$\begin{array}{cccccc} & \boxed{9} & \square & \square & \square \\ + & \boxed{1} & \boxed{0} & \square & \square \\ \hline \boxed{1} & \boxed{0} & \square & \square & \square \end{array}$$

So, till now $M = 1$, $S = 9$, O $= 0$. That means $0, 1, 9$ cannot be used further.

Next, here is a very interesting observation that at the hundred's place $E + 0 = N$. We know anything added to zero is the number itself, but here it didn't happen. That makes only one conclusion that is there must be a carry from one's.

$$\begin{array}{cccccc} & & \textcircled{1} & & \\ & \boxed{9} & \square & \square & \square \\ + & \boxed{1} & \boxed{0} & \square & \square \\ \hline \boxed{1} & \boxed{0} & \square & \square & \square \end{array}$$

So, simply $E + 1 = N$. But as of now, we don't have enough evidence to assign a value to E. So, we cannot assign any value to E right now.

Now, at ten's place $N + R = E$ is there. Let's make it more simple. We already figured out that $N = E + 1$.

$\therefore E + 1 + R = E$, but there might be a case of carry from the one's. Let's denote the possibility of 1 carry by $\textcircled{1}$. Further, we should keep in mind that there was a case of carrying at the hundred's place.

So, $E + 1 + R\textcircled{1} = E + 10 \Rightarrow 1 + R\textcircled{1} = 10 \Rightarrow R\textcircled{1} = 9$

We started with $\textcircled{1}$ as the possible carry case, but if nothing is being carried then R is supposed to be 9 which is not true because $S = 9$. So, the conclusion should be that $\textcircled{1}$ is carried over R.

$\therefore \boxed{R = 8}$

$$\begin{array}{cccccc} & & \textcircled{1} & \textcircled{1} & \\ & \boxed{9} & \square & \boxed{8} & \square \\ + & \boxed{1} & \boxed{0} & \square & \square \\ \hline \boxed{1} & \boxed{0} & \square & \square & \square \end{array}$$

We should list down the numbers that are yet to be assigned.

$$0, 1, 8, 9$$

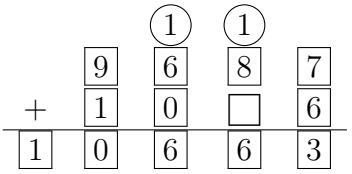
Now, $D + N = 10 + Y$ as it makes 1 carry in ten's. Let's check for how many cases it is possible.

$7+5$, $7+6$, $7+4$, $7+3$, $6+5$, $6+4$ these are possible combinations with remaining numbers. But from here a few cases should not be taken under consideration; firstly, which added up to 10 and 11, because if that happens then $Y = 1$ but we have already found that $M = 1, \mathrm{O} = 0$.

So the cases that will be taken under consideration are $7 + 5$, $7 + 6$.

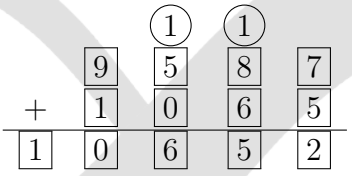
Let's check for $7 + 6$ ***case*** :

Let's put $D = 7$ and $E = 6$



We cannot complete this one. that means this one is not the required solution.

Let's check for $7 + 5$ ***case*** :



This is how we should solve a ***cryptarithmetic*** puzzle.

Note : This solution is not necessarily a unique solution of the puzzle. There might exist multiple possible solutions of a puzzle.

Vigenère cipher

The Vigenère cipher is a classic example of a polyalphabetic substitution cipher, meaning that it uses multiple substitution alphabets to encrypt and decrypt the message.

The Vigenère cipher is an improvement upon the simple Caesar cipher. While the Caesar cipher shifts all the letters in the maintext by a fixed number of positions, the Vigenère cipher uses a keyword to determine the specific number of positions each letter should be shifted.

The Vigenère cipher works by using a keyword to determine the shift value for each letter in the maintext. Let's walk through the steps of using the Vigenère cipher.

Encryption in Vigenère cipher

Method I : ***Using the direct approach***

Problem 3

Maintext : **Hello students**
Keyword : **Good**

Solution : Firstly, write down the *maintext* and the *keyword* like the following way.

H	E	L	L	O		S	T	U	D	E	N	T	S
G	O	O	D	G		O	O	D	G	O	O	D	G

Now we will assign a number to every alphabet serially from 0 to 25. Like the following.

A	B	C	D	E	F	G	H	I	J	K	L	M
0	1	2	3	4	5	6	7	8	9	10	11	12
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
13	14	15	16	17	18	19	20	21	22	23	24	25

To get the encrypted code of the maintext we need to add the serial number of the alphabet of the ***maintext*** to the corresponding alphabet of the ***keyword***, which would be the serial number of the encrypted alphabet.

For example, the serial numbers of $H \to 7$ and $G \to 6$;

So, the sum of the serial numbers $= 7 + 6 = 13 \to N$.

Only one thing that we should keep in mind is if the sum of the serial numbers gets more than 25 ***then the serial number for the encrypted code would be*** $(\boldsymbol{sum} - 26)$***.*** That means if the sum is 30 then the serial number for the encrypted code would be $30 - 26 = 4 \to E$

Now we have to solve it.

$E \to 4$ and O $\to 14$ So, $4 + 14 = 18 \to S$

$L \to 11$ and O $\to 14$ So, $11 + 14 = 25 \to Z$

$L \to 11$ and $D \to 3$ So, $11 + 3 = 14 \to$ O

O $\to 14$ and $G \to 6$ So, $14 + 6 = 20 \to U$

$S \to 18$ and O $\to 14$ So, $18 + 14 = 32 \Rightarrow (32 - 26) = 6 \to G$

$T \to 19$ and O $\to 14$ So, $19 + 14 = 33 \Rightarrow (33 - 26) = 7 \to H$

$U \to 20$ and $D \to 3$ So, $20 + 3 = 23 \to X$

$D \to 3$ and $G \to 6$ So, $3 + 6 = 9 \to J$

$E \to 4$ and O $\to 14$ So, $14 + 4 = 18 \to S$

$N \to 13$ and O $\to 14$ So, $13 + 14 = 27 \Rightarrow (27 - 26) = 1 \to B$

$T \to 19$ and $D \to 3$ So, $19 + 3 = 22 \to W$

$S \to 18$ and $G \to 6$ So, $18 + 6 = 24 \to Y$

So the encrypted message would be ***NSZOU GHXJSBWY***

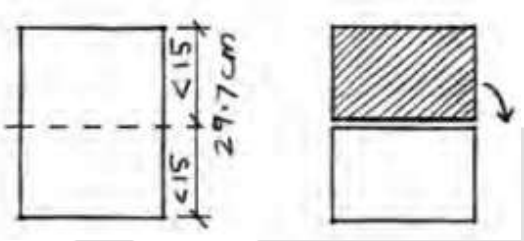
Maintext : **Hello students**
Keyword : **Good**
Encrypted text : **Nszou ghxjsbwy**

Maintext: **Solve The Questions**
Keyword: **Hard**

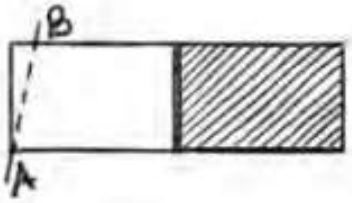
Activity I - 10 minutes

Making of a rectangle of breadth 15 cm from an A4 sheet

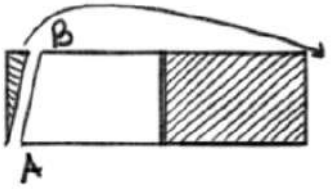
- First we have to take an A4 sheet. We know that the side of an A4 sheet is 29.7 so if we divide that length into two halves then both of them would be less than 15 cm and will place them one after one the way shown in the figure.



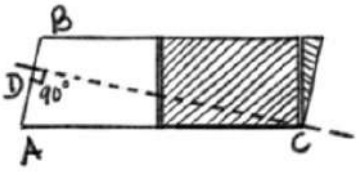
- Next we have to draw an exact 15 centimeters line from the bottom-left corner to the opposite side.



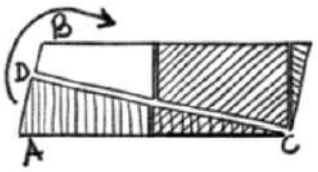
- Cut the paper from the drawn line and place it at the very right.



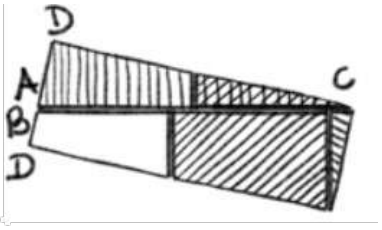
- Draw a perpendicular to BD from C.



- Again cut the paper through the perpendicular drawn and place it over them.



- The shape we are getting is a ***rectangle*** with 15 centimeters of breadth.



Activity II- 10 minutes

15 ×		3 ÷		4 -	2 ÷
2 ×	13 +				
	5 ÷		10 +		3 ×
30 ×		5 +	5 +		
1 -	36 ×		5 ÷	17 +	

Mixed Problems - 15 minutes

Problem 1

At the end of a game of marbles, Lei has 15 marbles, Dora has 8 and Omar has 4. How many marbles must Lei give back to his friends if they want to start the next game with an equal number each?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

Problem 2

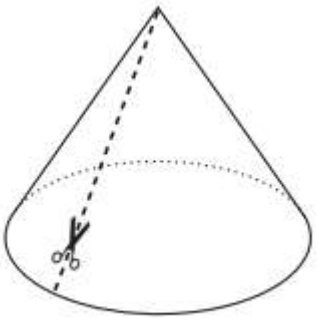
In the grid, the total of each row is given at the end of the row, and the total of each column is given at the bottom of the column.

The value of N is

8		6	16
	N		9
7		9	20
20	7	18	

Problem 3

At his birthday party, Ricky and his friends wear stripy paper hats in the shape of a cone, as shown on the left. After the party, Ricky makes a straight cut in one of the hats all the way up to the point at the top, as shown on the right.



Which of the following best matches what the hat will look like when Ricky flattens it out on the table?

(A) 

(B) 

(C) 

(D) 

(E) 

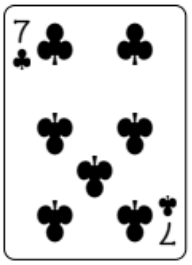
Problem 4

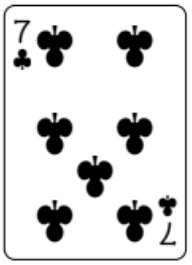
Emma is going to write all the numbers from 1 to 50 in order. She writes 25 digits on the first line of her page. What was the last number she wrote on this line?

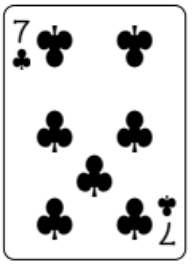
Problem 5

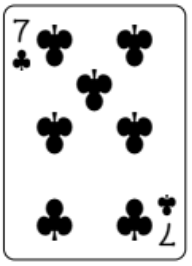
The playing card shown is flipped over along edge b and then flipped over again along edge c. What does it look like now?

$$270 \div 45 = \square \div 15$$

(A) 

(B) 

(C) 

(D) 

(E)